Question 13 **(10 marks)**

(a) Determine $\frac{d}{dx}(x^2 \ln x)$. (2 marks)

(b) Using your answer from part (a), show that the graph of $y = x^2 \ln x$ has only one stationary point. (3 marks)

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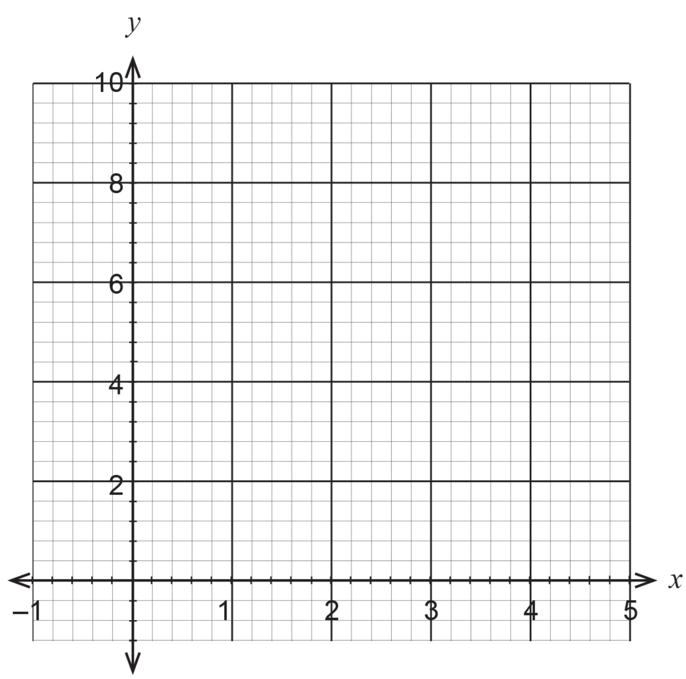
(c) Sketch the graph of $y = x^2 \ln x$, showing all features. (3 marks)



(d) Calculate the area bounded by the graph of $y = x^2 \ln x$, the x axis, $x = 1$ and $x = e$. (2 marks)

Question 6 **(7 marks)**

(a) Evaluate $\int_0^1 \frac{-12x}{1+3x^2}\,dx.$ (3 marks)

(b) Given $f(x) = \ln(2 - x^3)$

(i) determine $f'(1)$. (3 marks)

(ii) In relation to the graph of $f(x)$, explain the meaning of your answer to (b)(i). (1 mark)

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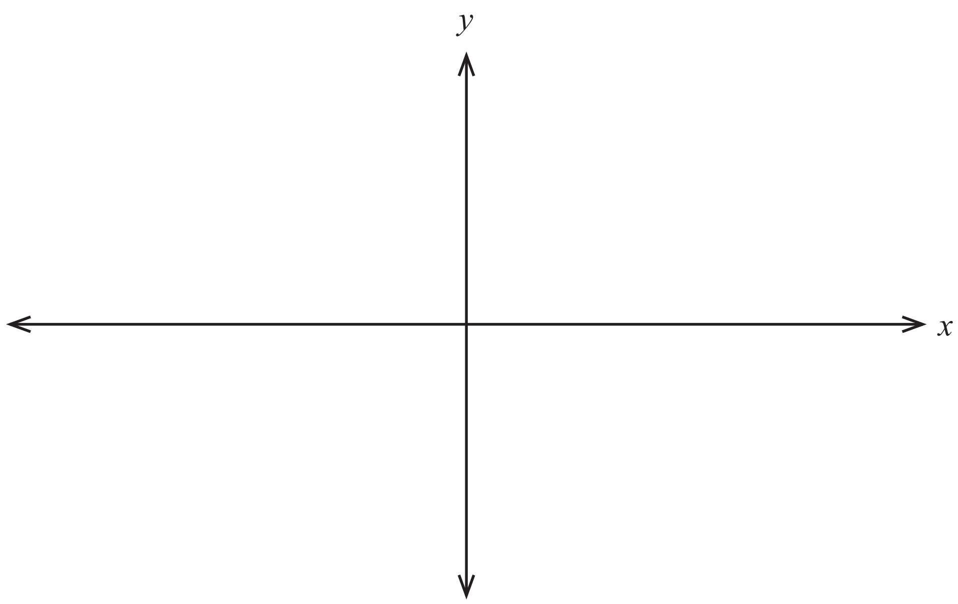
Question 14 **(9 marks)**

Let $f(x) = x \ln(x + 3)$.

(a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection. (5 marks)

(b) On the axes provided sketch the graph of $f(x)$, labelling all key features. (4 marks)

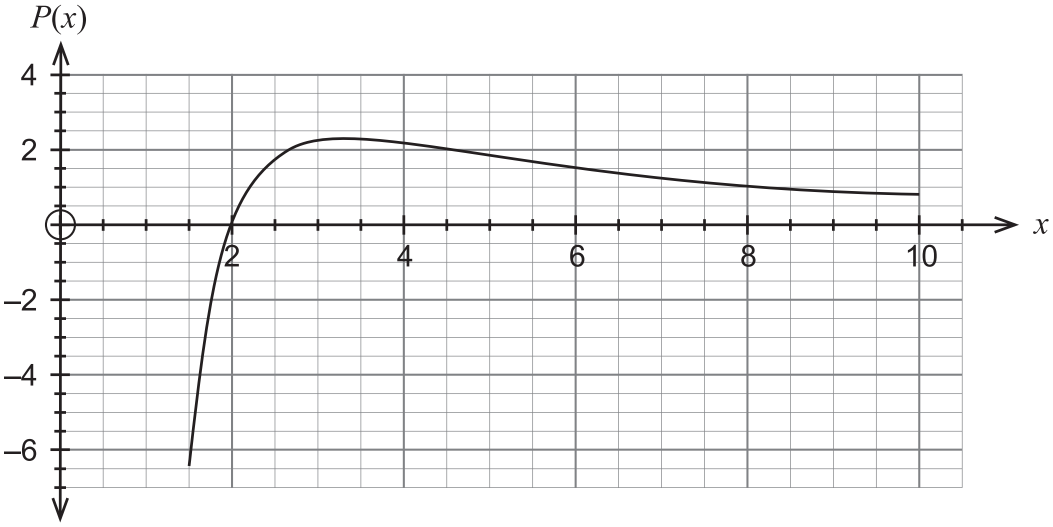


Question 6 **(8 marks)**

A company manufactures and sells an item for $\$x$. The profit, $\$P$, made by the company per item sold is dependent on the selling price and can be modelled by the function:

$$P(x) = \frac{50\ln\left(\frac{x}{2}\right)}{x^2} \text{ where } 1.5 \le x \le 10$$

The graph of $P(x)$ is shown below:



(a) Describe how the profit per item sold varies as the selling price changes. (3 marks)

(b) Determine the exact price that should be charged for the item if the company wishes to maximise the profit per item sold. (5 marks)

Question 7 **(10 marks)**

(a) Determine a simplified expression for $\frac{d}{dx}(x\ln(x))$. (2 marks)

(b) Use your answer from part (a) to show that $\int \ln(x)\,dx = x\ln(x) - x + c$, where c is a constant. (4 marks)

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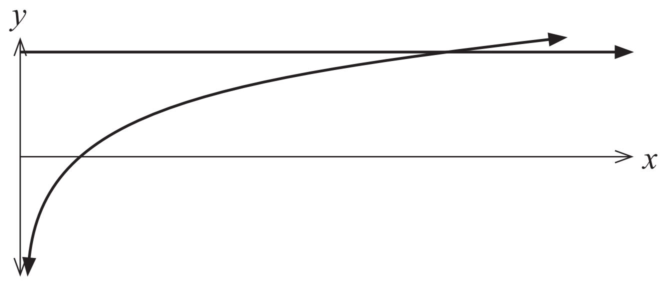
The graphs of the functions $f(x) = 5$ and $g(x) = \ln(x)$ are shown below.



(c) Determine the exact area enclosed between the x axis, the y axis and the functions $f(x)$ and $g(x)$. (4 marks)

End of section

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

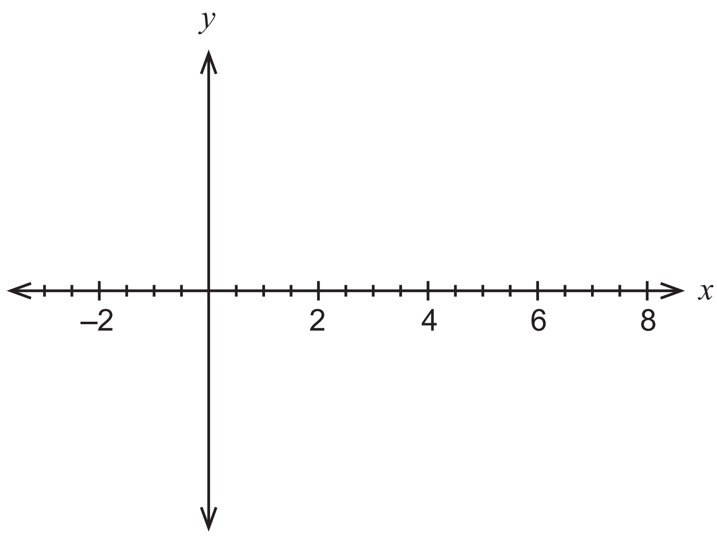
Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(8 marks)**

Consider the function $f(x) = \log_a (x - 1)$ where $a > 1$.

(a) On the axes below, sketch the graph of $f(x)$, labelling important features. (3 marks)



(b) Determine the value of m if $f(m) = 1$. (2 marks)

(c) Determine the coordinates of the x – intercept of $f(x + b) + c$, where b and c are positive real constants. (3 marks)

Question 18 **(7 marks)**

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, L, is given by the formula below:

$L = 10\log\left(\frac{I}{I_0}\right)$ dB where I is the sound intensity and I_0 is the reference sound intensity.

I and I_0 are measured in watt/m^2.

(a) Listening to a sound intensity of 5 billion times that of the reference intensity $(I = 5 \times 10^9 I_0)$ for more than 30 minutes is considered unsafe. To what sound intensity level does this correspond? (2 marks)

(b) The reference sound intensity, I_0, has a sound intensity level of 0 dB. If a household vacuum cleaner has a sound intensity $I = 1 \times 10^{-5}$ watt/m^2 and this corresponds to a sound intensity level $L = 70$ dB, determine I_0. (2 marks)

The average sound intensity level for rainfall is 50 dB and for heavy traffic 85 dB.

(c) How many times more intense is the sound of traffic than that of rainfall? (3 marks)

End of questions

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Question 6 **(7 marks)**

Consider the function $f(x) = \ln(x)$. The function $g(x) = f(x) + a$ is a vertical translation of f by a units.

(a) Express the function $g(x) = \ln(4x)$ in terms of a vertical translation of f (i.e. in the form $g(x) = f(x) + a$), stating the number of units that f is translated. (2 marks)

The function $h(x) = cf(x)$ is a vertical dilation of f by a scale factor of c.

(b) Express the function $h(x) = \ln(\sqrt{x})$ in terms of a vertical dilation of f, stating the scale factor. (2 marks)

The function $p(x) = f(bx)$ is a horizontal dilation of f by a scale factor of $\frac{1}{b}$.

(c) Express the function $p(x) = \ln(x) + 4$ in terms of a horizontal dilation of f, stating the scale factor. (3 marks)

Question 7 **(13 marks)**

Consider the function $f(x) = e^{2x} - 4e^{x}$.

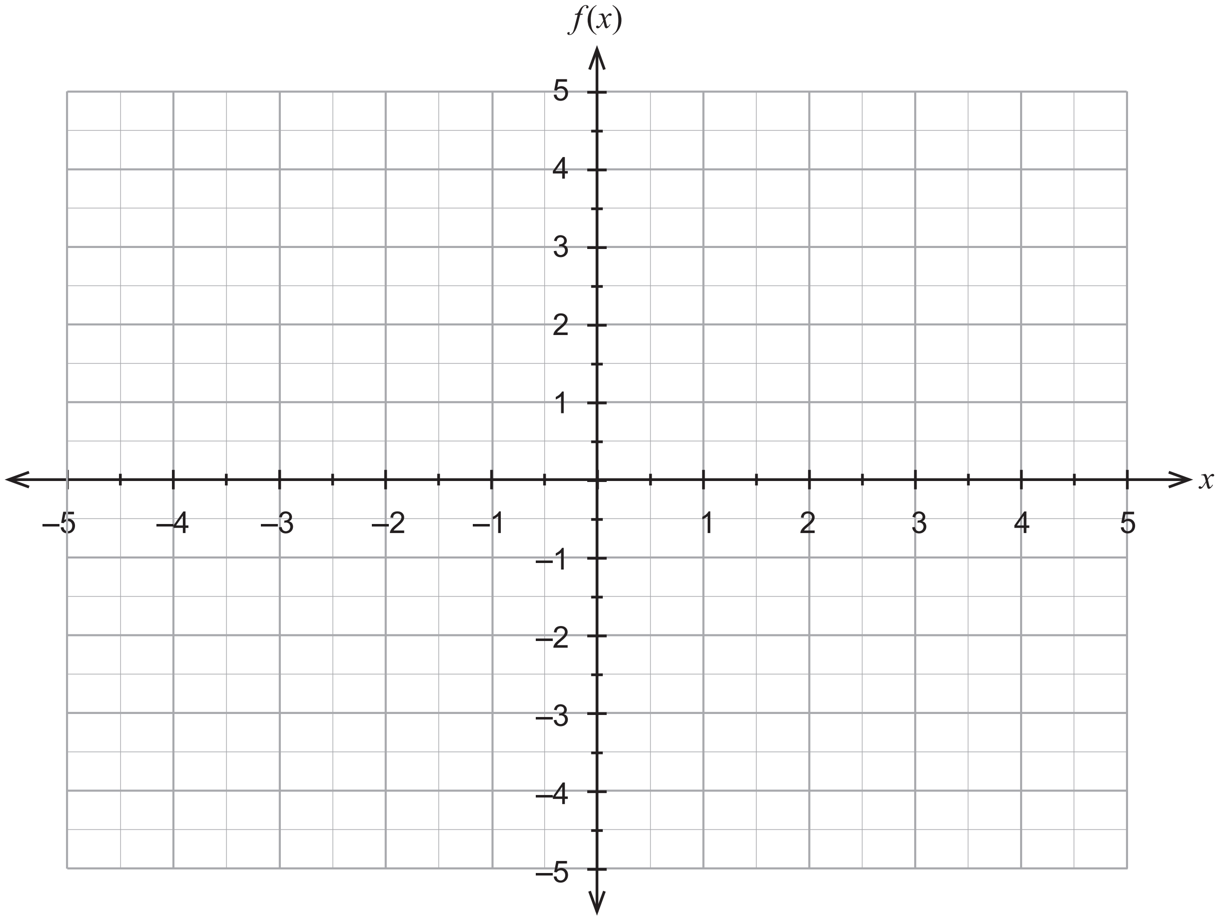
(a) Determine the coordinates of the x-intercept(s) of f. You may wish to consider the factorised version of $f: f(x) = e^{x}(e^{x} - 4)$. (3 marks)

(b) Show that there is only one turning point on the graph of f, which is located at $(\ln(2), -4)$. (3 marks)

(c) Determine the coordinates of the point(s) of inflection of f. (3 marks)

(d) Sketch the function f on the axes below, labelling clearly all intercepts, the turning point and point(s) of inflection. Some approximate values of the natural logarithmic function provided in the table below may be helpful. (4 marks)

x	1	2	3	4
$\ln(x)$	0	0.7	1.1	1.4



Question 10 **(7 marks)**

Water flows into a bowl at a constant rate. The water level, h, measured in centimetres, increases at a rate given by

$$h'(t) = \frac{4t + 1}{2t^2 + t + 1}$$

where the time t is measured in seconds.

(a) Determine the rate that the water level is rising when $t = 2$ seconds. (1 mark)

(b) Explain why $h(t) = \ln(2t^2 + t + 1) + c$. (2 marks)

(c) Determine the total change in the water level over the first 2 seconds. (1 mark)

The bowl is filled when the water level reaches $\ln(56)$ cm.

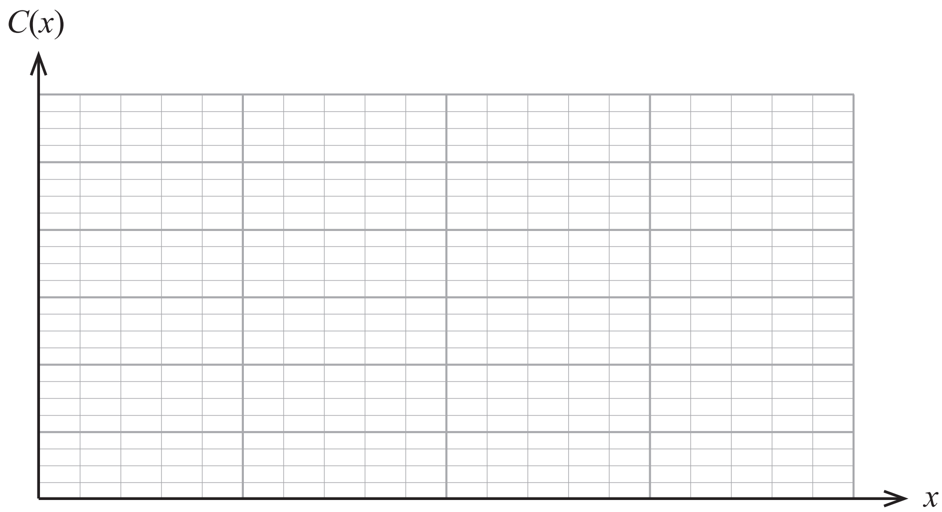
(d) If the bowl is initially empty, determine how long it takes for the bowl to be filled. (3 marks)

Question 13 **(7 marks)**

A company manufactures small machine components. They can manufacture up to 200 of a particular component in one day. The total cost, in hundreds of dollars, incurred in manufacturing the components is given by: $C(x) = \dfrac{x\ln(2x+1)}{3} - 2x + 120$, where x is the number of components that will be produced on that day.

(a) Determine the total cost of manufacturing 20 components in one day. (1 mark)

(b) On the axes below, sketch the graph of $C(x)$. (3 marks)



(c) With reference to your graph in part (b), explain how many components the company should manufacture per day if the total cost is to be as low as possible. (3 marks)

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Question 13 **(10 marks)**

(a) Determine $\frac{d}{dx}\left(x^2 \ln x\right)$. (2 marks)

Solution
$\frac{d}{dx}\left(x^2 \ln x\right) = x^2 \frac{1}{x} + \ln x(2x)$ $= x(1 + 2\ln x)$
Specific behaviours
✓ uses product rule ✓ determines derivative

(b) Using your answer from part (a), show that the graph of $y = x^2 \ln x$ has only one stationary point. (3 marks)

Solution
$\frac{dy}{dx} = x(1 + 2\ln x)$ $\frac{dy}{dx} = 0, \quad \ln x = -\frac{1}{2}, \quad x \neq 0$ Only one point where derivative is zero hence only one stationary point.
Specific behaviours
✓ equates derivative to zero ✓ states that $x \neq 0$ ✓ shows that only stationary point occurs for $\ln x = -\frac{1}{2}$

Question 6 **(7 marks)**

(a) Evaluate $\int_0^1 \frac{-12x}{1+3x^2}\,dx.$ (3 marks)

Solution
$\int_0^1 \frac{-12x}{1+3x^2}\,dx = -2\ln\left[1+3x^2\right]_0^1$ $= -2\left(\ln(4) - \ln(1)\right)$ $= \ln(\frac{1}{16})$ or $\{-\ln 16\}$ or $-2\ln 4$
Specific behaviours
✓ identifies the solution involving ln   ✓ determines correct expression   ✓ evaluates limits and correctly determines and simplifies solution

(b) Given $f(x) = \ln(2 - x^3)$

(i) determine $f'(1)$. (3 marks)

Solution
$\frac{dy}{dx} = \frac{-3x^2}{2-x^3}$ $\frac{dy}{dx}\Big\|_{x=1} = \frac{-3}{1}$ $= -3$
Specific behaviours
✓ identifies the need for the quotient rule to find derivative   ✓ correctly determines derivative   ✓ determines the derivative at x=1

(ii) In relation to the graph of $f(x)$, explain the meaning of your answer to (b)(i). (1 mark)

Solution
$f'(1)$ is the gradient of the curve (or the tangent to the curve) at the point where x=1
Specific behaviours
✓ explains meaning

Question 14 **(9 marks)**

Let $f(x) = x\ln(x+3)$.

(a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection. (5 marks)

Solution

$$\frac{df}{dx} = \frac{x}{x+3} + \ln(x+3)$$

For SPs: $0 = \dfrac{x}{x+3} + \ln(x+3)$

$$x = -1.1454$$

$$y = -0.7075$$

$$\frac{d^2 f}{dx^2} = \frac{x+3-x}{(x+3)^2} + \frac{1}{x+3} = \frac{x+6}{(x+3)^2}$$

$$\left.\frac{d^2 f}{dx^2}\right|_{x=-1.145449} = 1.411 = \text{positive}$$

therefore minimum

For POI: $\dfrac{d^2 f}{dx^2} = 0$

$\therefore$ POI when $x = -6$

So no POI as the function is undefined for $x \le -3$.

$\text{solve}\left(\ln(x+3) + \dfrac{x}{x+3} = 0, x\right)$

$\{x = -1.145449281\}$

$\dfrac{d}{dx}\left(\ln(x+3) + \dfrac{x}{x+3}\right)$

$\dfrac{x+6}{(x+3)^2}$

$\dfrac{d^2}{dx^2}(f(x))\,|\,x=-1.145449281$

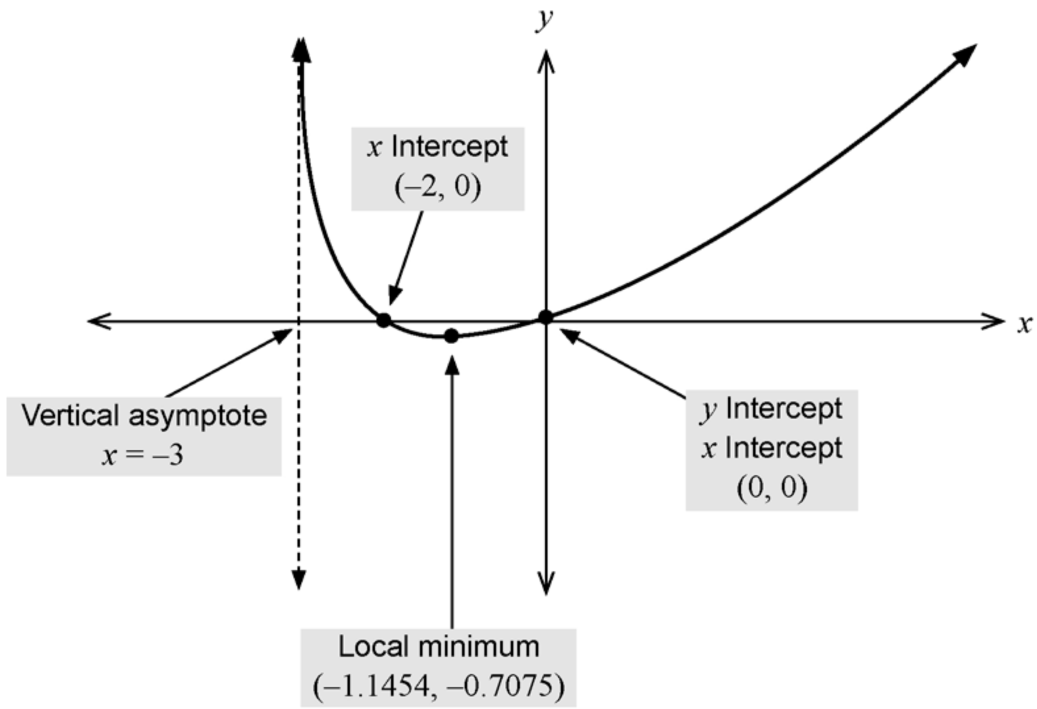
1.411469872

Specific behaviours

✓ differentiates correctly
✓ finds the critical point
✓ finds y co-ordinate and justifies minimum
✓ finds second derivative
✓ rejects point of inflection

Question 14 (continued)

(b) On the axes provided sketch the graph of $f(x)$, labelling all key features. (4 marks)

Solution

Specific behaviours
✓ shows vertical asymptote ✓ shows minimum ✓ shows intercepts ✓ has correct shape of graph

Question 17 **(6 marks)**

A beverage company has decided to release a new product. 'Joosilicious' is to be sold in 375 mL cans that are perfectly cylindrical. {Hint: $1mL = 1cm^3$}

(a) If the cans have a base radius of x cm show that the surface area of the can, S, is given by: $S = 2\pi x^2 + \frac{750}{x}$. (2 marks)

Solution
$375 = \pi x^2 h$ $\therefore h = \frac{375}{\pi x^2}$ $S = 2\pi x^2 + 2\pi x h$ $= 2\pi x^2 + 2\pi x\left(\frac{375}{\pi x^2}\right)$ $= 2\pi x^2 + \frac{750}{x}$
Specific behaviours
✓ uses volume formula to determine h in terms of x ✓ demonstrates substitution of h into surface area formula and simplifies to show result

(b) Using calculus methods, and showing full reasoning and justification, find the dimensions of the can that will minimise its surface area. (4 marks)

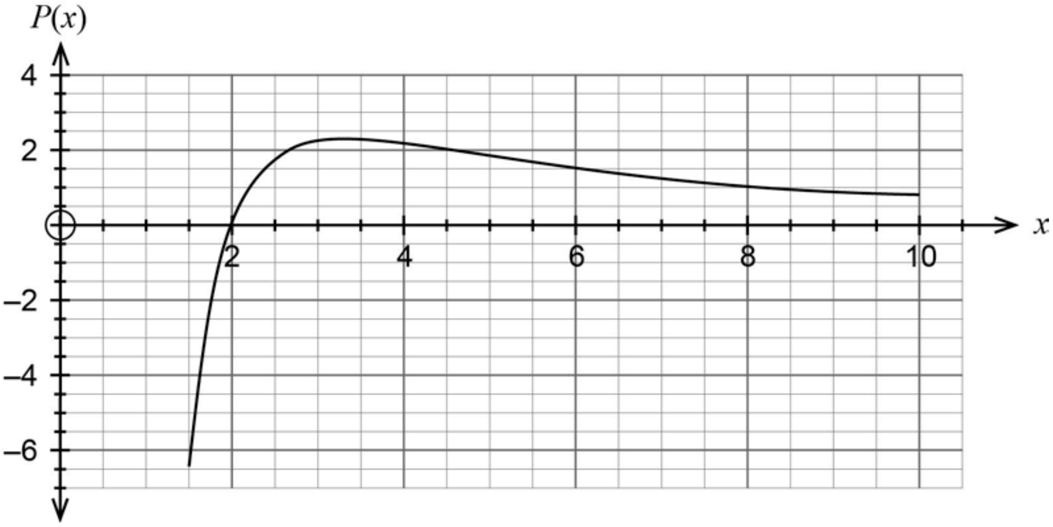
Solution
$S = 2\pi x^2 + \frac{750}{x}$ $\frac{dS}{dx} = 4\pi x - \frac{750}{x^2}$ $0 = 4\pi x - \frac{750}{x^2}$ $x = 3.908$ cm $\frac{d^2S}{dx^2} = 4\pi + \frac{1500}{x^3}$ $\frac{d^2S}{dx^2}\Big\rvert_{x=3.908} = +ve\ (37.7) \Rightarrow \text{Min}$ When $x = 3.908$, $h = 7.816$ Cans have a radius of 3.9 cm and a height of 7.8 cm to minimise surface area
Specific behaviours
✓ determines first derivate ✓ equates to zero to find x ✓ justifies minimum with second derivative or other suitable method ✓ states dimensions of can

Question 6 **(8 marks)**

A company manufactures and sells an item for $\$x$. The profit, $\$P$, made by the company per item sold is dependent on the selling price and can be modelled by the function:

$$P(x) = \frac{50\ln\left(\frac{x}{2}\right)}{x^2} \quad \text{where } 1.5 \le x \le 10 .$$

The graph of $P(x)$ is shown below:



(a) Describe how the profit per item sold varies as the selling price changes. (3 marks)

Solution
The company will make a loss for a selling price between \$1.50 and \$2.00. The profit then increases up to approximately \$2.25 per item sold for a selling price of approximately \$3.25, and then decreases steadily to a value of less than \$1 per item sold for a selling price of \$10.
Specific behaviours
✓ states initially making a loss ✓ states profit increases to maximum at \$3.25 ✓ states it decreases after that

Question 6 (continued)

(b) Determine the exact price that should be charged for the item if the company wishes to maximise the profit per item sold. (5 marks)

Solution
$\frac{dP}{dx} = \frac{x^2\left(\frac{50}{2}\times\frac{2}{x}\right) - 2x\times 50\ln\left(\frac{x}{2}\right)}{x^4}$ $= \frac{50x - 100x\ln\left(\frac{x}{2}\right)}{x^4}$ $= \frac{50 - 100\ln\left(\frac{x}{2}\right)}{x^3}$ For max, $\frac{dP}{dx} = 0 \Rightarrow 0 = \frac{50 - 100\ln\left(\frac{x}{2}\right)}{x^3}$ $\ln\left(\frac{x}{2}\right) = \frac{1}{2}$ $x = 2e^{\frac{1}{2}}$
Specific behaviours
✓ correctly states the numerator of the quotient rule ✓ correctly states the denominator of the quotient rule ✓ simplifies derivative ✓ equates simplified derivative to zero ✓ determines exact value of x

Question 7 **(10 marks)**

(a) Determine a simplified expression for $\frac{d}{dx}\left(x\ln(x)\right)$. (2 marks)

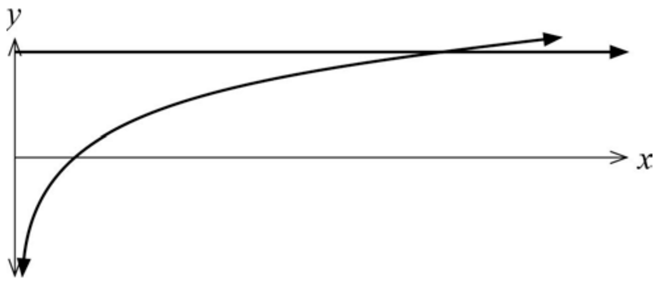
Solution
$\frac{d}{dx}\left(x\ln(x)\right) = x \times \frac{1}{x} + \ln\left(x\right)$ $= 1 + \ln\left(x\right)$
Specific behaviours
✓ uses product rule to determine derivative ✓ simplifies the derivative

(b) Use your answer from part (a) to show that $\int \ln(x)dx = x\ln(x) - x + c$, where c is a constant. (4 marks)

Solution
$\frac{d}{dx}\left(x\ln(x)\right) = 1 + \ln\left(x\right)$ $\int \frac{d}{dx}\left(x\ln(x)\right)\,dx = \int\left(1 + \ln\left(x\right)\right)\,dx$ $x\ln(x) = x + \int \ln\left(x\right)\,dx + c$ $\int \ln\left(x\right)\,dx = x\ln(x) - x + c$
Specific behaviours
✓ integrates both sides of answer from part (a) ✓ partly integrates right-hand side to get x ✓ uses fundamental theorem of calculus to simplify the left-hand side ✓ rearranges to give the required result

Question 7 (continued)

The graphs of the functions $f(x)=5$ and $g(x)=\ln(x)$ are shown below.



(c) Determine the exact area enclosed between the x-axis, the y-axis and the functions $f(x)$ and $g(x)$. (4 marks)

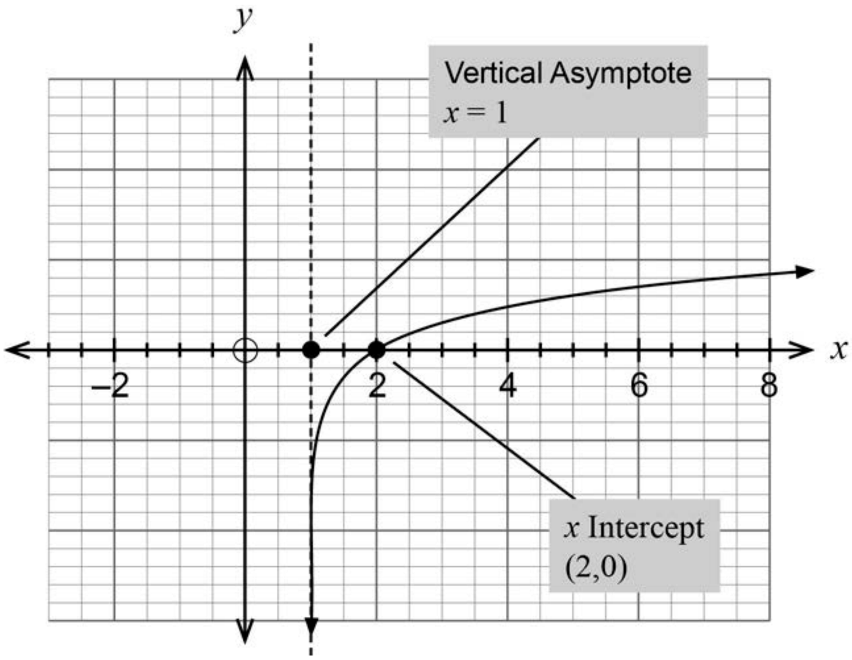
Solution
Intersect when: $\ln(x)=5 \Rightarrow x=e^5$ Area under $f(x)$: $\int_1^{e^5} \ln(x)dx = [x\ln(x)-x]_1^{e^5}$ $=5e^5-e^5+1$ Required area $=5\times e^5-(5e^5-e^5+1)$ $=e^5-1$
Specific behaviours
✓ determines point of intersection between $f(x)$ and $g(x)$ ✓ states an integral for the area under $f(x)$ ✓ evaluates integral ✓ determines required area

Section Two: Calculator-assumed **65% (99 Marks)**

Question 8 **(8 marks)**

Consider the function $f(x) = \log_a(x-1)$ where $a > 1$.

(a) On the axes below, sketch the graph of $f(x)$, labelling important features. (3 marks)



Solution
See graph
Specific behaviours
✓ asymptote at $x = 1$ ✓ gives correct shape ✓ x-int at $x = 2$

(b) Determine the value of m if $f(m) = 1$. (2 marks)

Solution
$1 = \log_a(m-1)$ $m - 1 = a$ $m = a + 1$
Specific behaviours
✓ equates $f(m)$ to 1 ✓ solves for m

(c) Determine the coordinates of the x – intercept of $f(x+b)+c$, where b and c are positive real constants. (3 marks)

Solution
$0 = \log_a(x-1+b)+c$ $-c = \log_a(x-1+b)$ $a^{-c} = x-1+b$ $x = a^{-c}+1-b$ coordinates are: $\left(a^{-c}+1-b, 0\right)$
Specific behaviours
✓ equates new function to zero ✓ solves for x ✓ states coordinates

Question 18 **(7 marks)**

The ear has the remarkable ability to handle an enormous range of sound levels. In order to express levels of sound meaningfully in numbers that are more manageable, a logarithmic scale is used, rather than a linear scale. This scale is the decibel (dB) scale.

The sound intensity level, L, is given by the formula below:

$L = 10\log\left(\frac{I}{I_0}\right)$ dB where I is the sound intensity and I_0 is the reference sound intensity.

I and I_0 are measured in watt/m^2.

(a) Listening to a sound intensity of 5 billion times that of the reference intensity ($I = 5\times10^9 I_0$) for more than 30 minutes is considered unsafe. To what sound intensity level does this correspond? (2 marks)

Solution
$L = 10\log\left(\frac{5\times10^9 I_0}{I_0}\right)$ $\approx 97\text{ dB}$
Specific behaviours
✓ substitutes for L ✓ calculates level

(b) The reference sound intensity, I_0, has a sound intensity level of 0 dB. If a household vacuum cleaner has a sound intensity, $I = 1\times10^{-5}\text{ watt/m}^2$ and this corresponds to a sound intensity level $L = 70$ dB, determine I_0. (2 marks)

Solution
$70 = 10\log\left(\frac{1\times10^{-5}}{I_0}\right)$ $I_0 = \frac{1\times10^{-5}}{10^7} = 1\times10^{-12}\text{ watt/m}^2$
Specific behaviours
✓ substitutes for L and I ✓ determines I_0 including units

The average sound intensity level for rainfall is 50 dB and for heavy traffic 85 dB.

(c) How many times more intense is the sound of traffic than that of rainfall? (3 marks)

Solution
$50 = 10\log\left(\frac{I_{rain}}{I_0}\right) \Rightarrow \frac{I_{rain}}{I_0} = 10^5 \Rightarrow I_{rain} = 10^5 I_0$ $85 = 10\log\left(\frac{I_{traffic}}{I_0}\right) \Rightarrow \frac{I_{traffic}}{I_0} = 10^{8.5} \Rightarrow I_{traffic} = 10^{8.5} I_0$ $\therefore \frac{I_{traffic}}{I_{rain}} = \frac{10^{8.5}}{10^5} = 10^{3.5} \approx 3200$
Specific behaviours
✓ rearranges logarithmic equations to exponentials ✓ writes ratio and cancels I_0 ✓ determines how many more times intense

Question 6 **(7 marks)**

Consider the function $f(x) = \ln(x)$. The function $g(x) = f(x) + a$ is a vertical translation of f by a units.

(a) Express the function $g(x) = \ln(4x)$ in terms of a vertical translation of f (i.e. in the form $g(x) = f(x) + a$), stating the number of units that f is translated. (2 marks)

Solution
$g(x) = \ln(4x)$ $= \ln(4) + \ln(x)$ $= f(x) + \ln(4)$ f is translated vertically (upward) by $\ln(4)$ units.
Specific behaviours
✓ expresses $g(x)$ as a sum of logs ✓ recognises a vertical translation by $\ln(4)$ units

The function $h(x) = cf(x)$ is a vertical dilation of f by a scale factor of c.

(b) Express the function $h(x) = \ln(\sqrt{x})$ in terms of a vertical dilation of f, stating the scale factor. (2 marks)

Solution
$h(x) = \ln(\sqrt{x})$ $= \ln(x^{0.5})$ $= 0.5\ln(x)$ $= 0.5f(x)$ f is scaled vertically by a factor of 0.5.
Specific behaviours
✓ expresses h as a product involving $\ln(x)$ ✓ recognises a vertical scaling by a scale factor of 0.5

The function $p(x) = f(bx)$ is a horizontal dilation of f by a scale factor of $\frac{1}{b}$.

(c) Express the function $p(x) = \ln(x) + 4$ in terms of a horizontal dilation of f, stating the scale factor. (3 marks)

Solution
$p(x) = \ln(x) + 4$ $= \ln(x) + 4\ln(e)$ $= \ln(x) + \ln(e^4)$ $= \ln(e^4x)$ $= f(e^4x)$ f is scaled horizontally by a scale factor of e^{-4}.
Specific behaviours
✓ expresses 4 as $4\ln(e)$ ✓ expresses p using a single logarithm ✓ states horizontal scale factor

Question 7 **(13 marks)**

Consider the function $f(x) = e^{2x} - 4e^{x}$.

(a) Determine the coordinates of the x-intercept(s) of f. You may wish to consider the factorised version of $f : f(x) = e^{x}(e^{x} - 4)$. (3 marks)

Solution
Solve $f(x) = 0$ $0 = e^{x}(e^{x} - 4)$ $e^{x} = 4$ $x = \ln(4)$ Hence x-intercept at $(\ln(4), 0)$.
Specific behaviours
✓ states correct equation to be solved ✓ solves for x ✓ states coordinates

(b) Show that there is only one turning point on the graph of f, which is located at $(\ln(2), -4)$. (3 marks)

Solution
$f'(x) = 2e^{2x} - 4e^{x}$ Solve $f'(x) = 0$ $0 = 2e^{2x} - 4e^{x}$ $= 2e^{x}(e^{x} - 2)$ $e^{x} = 2$ $x = \ln(2)$ Substitute $x = \ln(2)$ into $f(x)$ $f(\ln(2)) = e^{2\ln(2)} - 4e^{\ln(2)}$ $= e^{\ln(4)} - 4e^{\ln(2)}$ $= 4 - 8$ $= -4$ Turning point at $(\ln(2), -4)$.
Specific behaviours
✓ differentiates $f(x)$ correctly and equates to 0 ✓ shows the steps required to solve for x ✓ demonstrates the use of log laws to determine the y-coordinate

Question 7 (continued)

(c) Determine the coordinates of the point(s) of inflection of f. (3 marks)

Solution
$f''(x) = 4e^{2x} - 4e^{x}$ Solve $f''(x) = 0$ $0 = 4e^{2x} - 4e^{x}$ $= 4e^{x}(e^{x} - 1)$ $e^{x} = 1$ $x = \ln(1) = 0$ Substitute $x = 0$ into $f(x)$ $f(\ln(2)) = e^{2(0)} - 4e^{0}$ $= 1 - 4$ $= -3$ Inflection point at $(0, -3)$.
Specific behaviours
✓ differentiates $f'(x)$ correctly and equates to 0   ✓ solves for x ✓ determines y-coordinate of inflection point

Question 10 **(7 marks)**

Water flows into a bowl at a constant rate. The water level, h, measured in centimetres, increases at a rate given by

$$h'(t) = \frac{4t+1}{2t^2+t+1}$$

where the time t is measured in seconds.

(a) Determine the rate that the water level is rising when $t = 2$ seconds. (1 mark)

Solution
$h'(2) = \frac{4(2)+1}{2(2)^2+(2)+1}$ $= \frac{9}{11}$ cm/s $\{0.818\}$
Specific behaviours
✓ determines correct rate including units

(b) Explain why $h(t) = \ln(2t^2+t+1) + c$. (2 marks)

Solution
$h'(t)$ is of the form $\frac{f'(x)}{f(x)}$ (the numerator is the derivative of the denominator), so the function $h(t)$ is the natural logarithm of the denominator.   Also, +c needs to be included in the function, as any constant could be included here.
Specific behaviours
✓ states that the numerator is the derivative of the denominator   ✓ identifies the number c as the constant of integration

(c) Determine the total change in the water level over the first 2 seconds. (1 mark)

Solution
$\Delta h = \int_0^2 \frac{4t+1}{2t^2+t+1}\,dt$ $= \ln(11)$ cm $\{2.398\}$
Specific behaviours
✓ determines total change

The bowl is filled when the water level reaches $\ln(56)$ cm.

(d) If the bowl is initially empty, determine how long it takes for the bowl to be filled. (3 marks)

Solution
Let the time taken for the bowl to be filled = a seconds $\ln(56) = \int_0^a \frac{4t+1}{2t^2+t+1}\,dt$ $= \left[\ln\left(2t^2+t+1\right)\right]_0^a$ $= \ln\left(2a^2+a+1\right)$ $56 = 2a^2+a+1$ $a = 5$ The bowl will take 5 seconds to completely fill.
Specific behaviours
✓ states a definite integral for depth of water ✓ equates definite integral to maximum water level ✓ determines time taken

Question 13 **(7 marks)**

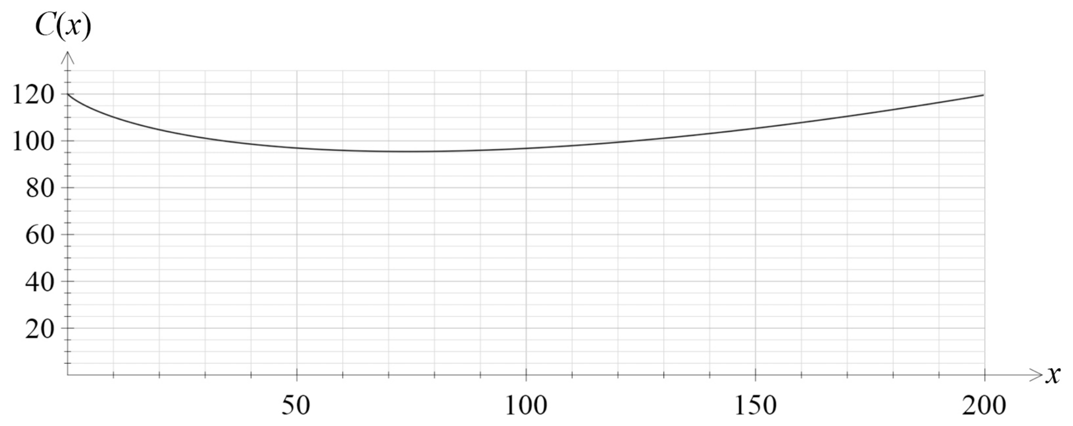
A company manufactures small machine components. They can manufacture up to 200 of a particular component in one day. The total cost, in hundreds of dollars, incurred in manufacturing the components is given by: $C(x)=\frac{x\ln(2x+1)}{3}-2x+120$, where x is the number of components that will be produced on that day.

(a) Determine the total cost of manufacturing 20 components in one day. (1 mark)

Solution
$C(20)=\frac{20\ln(41)}{3}-40+120=104.7571$ i.e. $\$10475.71\approx\$10476$
Specific behaviours
✓ determines the correct cost

(b) On the axes below, sketch the graph of $C(x)$. (3 marks)

Solution



Specific behaviours
✓ graph covers correct domain
✓ $C(0)$ and $C(200)$ are correct
✓ minimum between 70 and 80

(c) With reference to your graph in part (b), explain how many components the company should manufacture per day if the total cost is to be as low as possible. (3 marks)

Solution
The minimum is at $x=74.205$ $C(74)=95.4307$ i.e. \$9543.07 $C(75)=95.4320$ i.e. \$9543.20 The company should manufacture 74 components
Specific behaviours
✓ states graph is a minimum at x =74.205
✓ determines cost values for x =74 and x =75
✓ states that 74 components should be manufactured per day